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A Level Mathematics A

H240/01 Pure Mathematics
Sample Question Paper

Date – Morning/Afternoon

Version 2.1

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \dots \ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \dots \ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \dots \ + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \ \dots \qquad \left(|x|<1,\ \ n \in \mathbb{R}\right)$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1-\frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \textbf{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \le z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

1 Solve the simultaneous equations.

$$\begin{aligned} x^2+8x+y^2 &= 84 \\ x-y &= 10 \end{aligned}$$ **[4]**

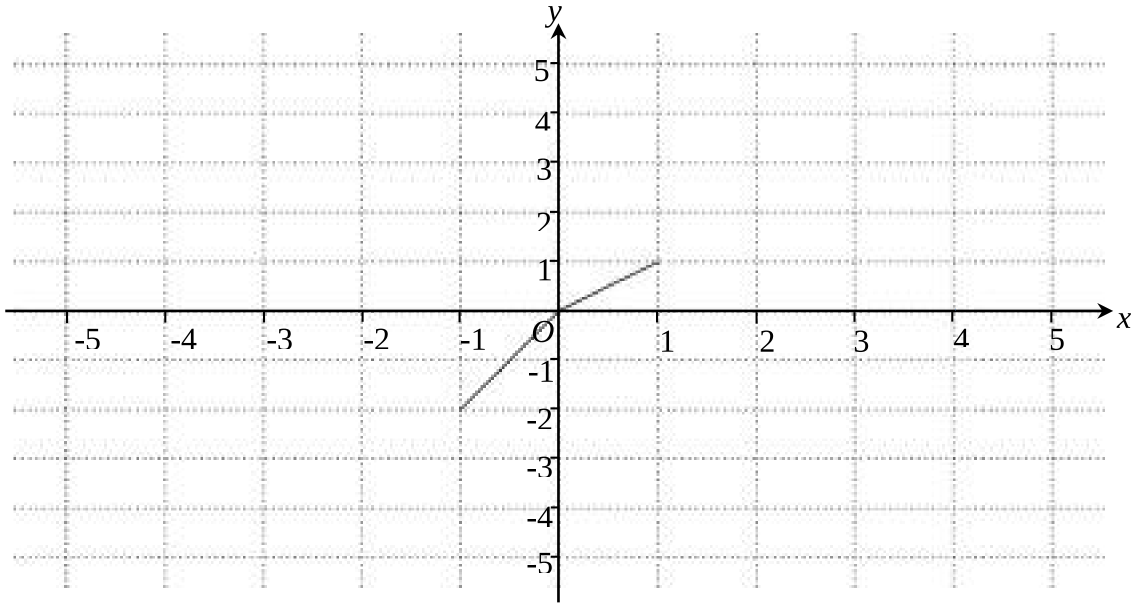
2 The points A, B and C have position vectors $3\mathbf{i}-4\mathbf{j}+2\mathbf{k}$, $-\mathbf{i}+6\mathbf{k}$ and $7\mathbf{i}-4\mathbf{j}-2\mathbf{k}$ respectively.
M is the midpoint of BC.

(a) Show that the magnitude of $\overrightarrow{OM}$ is equal to $\sqrt{17}$. **[2]**

Point D is such that $\overrightarrow{BC}=\overrightarrow{AD}$.

(b) Show that position vector of the point D is $11\mathbf{i}-8\mathbf{j}-6\mathbf{k}$. **[3]**

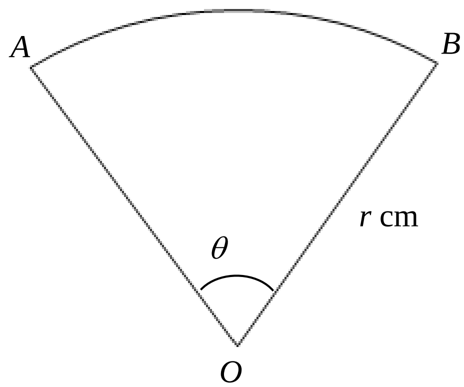
3 The diagram below shows the graph of $y = \mathrm{f}(x)$.



(a) On the diagram in the Printed Answer Booklet, draw the graph of $y = \mathrm{f}(\frac{1}{2}x)$. **[1]**

(b) On the diagram in the Printed Answer Booklet, draw the graph of $y = \mathrm{f}(x-2)+1$. **[2]**

4 The diagram shows a sector AOB of a circle with centre O and radius r cm.



The angle AOB is θ radians. The arc length AB is 15 cm and the area of the sector is 45 cm^2.

(a) Find the values of r and θ . **[4]**

(b) Find the area of the segment bounded by the arc AB and the chord AB. **[3]**

5 **In this question you must show detailed reasoning.**

Use logarithms to solve the equation $3^{2x+1} = 4^{100}$, giving your answer correct to 3 significant figures. **[4]**

6 Prove by contradiction that there is no greatest even positive integer. **[3]**

7 Business A made a £5000 profit during its first year.
In each subsequent year, the profit increased by £1500 so that the profit was £6500 during the second year, £8000 during the third year and so on.

Business B made a £5000 profit during its first year.
In each subsequent year, the profit was 90% of the previous year's profit.

(a) Find an expression for the total profit made by business A during the first *n* years. Give your answer in its simplest form. **[2]**

(b) Find an expression for the total profit made by business B during the first *n* years. Give your answer in its simplest form. **[3]**

(c) Find how many years it will take for the total profit of business A to reach £385 000. **[3]**

(d) Comment on the profits made by each business in the long term. **[2]**

8 **(a)** Show that $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$. **[3]**

(b) **In this question you must show detailed reasoning.**

Solve $\dfrac{2\tan\theta}{1+\tan^2\theta} = 3\cos 2\theta$ for $0 \le \theta \le \pi$. **[3]**

9 The equation $x^3 - x^2 - 5x + 10 = 0$ has exactly one real root α.

(a) Show that the Newton-Raphson iterative formula for finding this root can be written as

$$x_{n+1} = \frac{2x_n^{\ 3} - x_n^{\ 2} - 10}{3x_n^{\ 2} - 2x_n - 5}.$$ **[3]**

(b) Apply the iterative formula in part (a) with initial value $x_1 = -3$ to find x_2, x_3, x_4 correct to 4 significant figures. **[1]**

(c) Use a change of sign method to show that $\alpha = -2.533$ is correct to 4 significant figures. **[3]**

(d) Explain why the Newton-Raphson method with initial value $x_1 = -1$ would not converge to α. **[2]**

10 A curve has equation $x = (y+5)\ln(2y-7)$.

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **[3]**

(b) Find the gradient of the curve where it crosses the y-axis. **[5]**

11 For all real values of *x*, the functions f and g are defined by $\mathrm{f}(x) = x^2 + 8ax + 4a^2$ and $\mathrm{g}(x) = 6x - 2a$, where *a* is a positive constant.

(a) Find $\mathrm{fg}(x)$.
Determine the range of $\mathrm{fg}(x)$ in terms of *a*. **[4]**

(b) If $\mathrm{fg}(2) = 144$, find the value of *a*. **[3]**

(c) Determine whether the function fg has an inverse. **[2]**

12 The parametric equations of a curve are given by $x = 2\cos\theta$ and $y = 3\sin\theta$ for $0 \le \theta < 2\pi$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. **[2]**

The tangents to the curve at the points P and Q pass through the point (2, 6).

(b) Show that the values of θ at the points P and Q satisfy the equation $2\sin\theta + \cos\theta = 1$. **[4]**

(c) Find the values of θ at the points *P* and *Q*. **[5]**

13 **In this question you must show detailed reasoning.**

Find the exact values of the x-coordinates of the stationary points of the curve $x^3 + y^3 = 3xy + 35$.

[9]

14 John wants to encourage more birds to come into the park near his house.

Each day, starting on day 1, he puts bird food out and then observes the birds for one hour. He records the maximum number of birds that he observes at any given moment in the park each day.

He believes that his observations may be modelled by the following differential equation, where n is the maximum number of birds that he observed at any given moment on day t.

$$\frac{\mathrm{d}n}{\mathrm{d}t} = 0.1n\left(1 - \frac{n}{50}\right)$$

(a) Show that the general solution to the differential equation can be written in the form $n = \dfrac{50A}{\mathrm{e}^{-0.1t} + A}$, where A is an arbitrary positive constant. **[9]**

(b) Using his model, determine the maximum number of birds that John would expect to observe at any given moment in the long term. **[1]**

(c) Write down one possible refinement of this model. **[1]**

(d) Write down one way in which John's model is not appropriate. **[1]**

END OF QUESTION PAPER